

SPIN-PARITY EFFECT IN VIOLATION OF BELL'S INEQUALITIES FOR ENTANGLED STATES OF PARALLEL POLARIZATION

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Bell inequalities (BIs) derived in terms of quantum probability statistics are extended to general bipartite-entangled states of arbitrary spins with parallel polarization. The original formula of Bell for the two-spin singlet is slightly modified in the parallel configuration, while, the inequality formulated by Clauser-Horne-Shimony-Holt remains not changed. The violation of BIs indeed resulted from the quantum non-local correlation for spin-1/2 case. However, the inequalities are always satisfied for the spin-1 entangled states regardless of parallel or antiparallel polarizations of two spins. The spin parity effect originally demonstrated with the antiparallel spin-polarizations (Mod. Phys. Lett. B28, 145004) still exists for the parallel case. The quantum non-locality does not lead to the violation for integer spins due to the cancellation of non-local interference effects by the quantum statistical-average. Again the violation of BIs seems a result of the measurement induced nontrivial Berry-phase for half-integer spins.

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1. Introduction

Non-locality being the most peculiar characteristic of quantum mechanics seems inconsistency with the classical field-theory based on the relativistic causality. Quantum entanglement as one of the most striking feature of quantum mechanics has become a resource for quantum computing and quantum information with technological breakthroughs in these areas.¹⁻³ Bell's theorem known as Bell's inequality (BI) proved that the existence of entangled quantum states has no classical counterpart and therefore provides a possibility of quantitative test for non-local correlations. BI was originally derived under classical statistics with the assumption of "locality", which means that physical systems cannot be instantly affected by distant objects in a space-like distance. Various extensions of the original BI have investigated^{4, 5} and attracted considerable attentions both theoretically and experimentally.^{1, 2, 6-10} The violation of BIs is predicted in some particular entangled-states with overwhelming experimental evidence,^{5-9, 11-15} which invalidates local realistic interpretations of quantum mechanics. The profound application of non-locality is mainly in the quantum information with a space-time separation not achievable for the classical systems. The violation of BIs constitutes an ability to faithfully produce, control and read out entangled states of qubit-pairs. The non-locality now has become a powerful resource^{2, 10} of quantum information science.¹⁶⁻²⁴ Nevertheless, there are still many open problems in relation with both bipartite and multipartite entanglements. Although the experimental evidence seems strongly support the non-local nature the underlying physical-principle is obscure²⁵ and many aspects in relation with the initial debate remain to be fully understood. The non-determinism and non-locality have been bearing continuously theoretical scrutiny ever since the birth of quantum mechanics.

To understand underlying physical principle of the violation of BIs we in a previous paper²⁶ adopted the density operators of entangled states to evaluate the measurement-outcome-correlations in terms of quantum probability-statistics. With the separation of density operators into local and non-local parts the non-local outcome-correlations are obtained explicitly. The local part of density operator describes the classical probability-statistics in the absence of quantum interference between two components of the entangled state. The BI is verified from the local part of density

operator²⁷ alone with the assumption of measurement-outcome-independence. On the other hand the violation of BIs indeed resulted from non-local correlations of entangled states. A spin parity effect in the violation of BIs was predicted²⁶ that BIs are violated by the half-integer spin entangled-states but not the integer spins. Moreover the violation in entangled spin-states is seen to be an effect of Berry phase (BP) induced by relative-reversal measurements of two spins. We established for the first time a relation between two kinds of non-localities, namely, the violation of BI violation and the so-called dynamic non-locality regarded as the geometric phase interference of quantum states.²⁶ The spin-parity phenomenon was originally demonstrated in terms of a spin singlet following the model of Bell. It is a natural question that whether or not the spin-parity effect exists only for these particular states with antiparallel spin-polarizations. We in the present paper consider bipartite-entangled state with parallel spin-polarization, which leads to the modification of original form of BI. However the inequality derived by Clauser-Horne-Shimony-Holt (CHSH) remains not changed. The spin-parity phenomenon for the violation of BIs seems independent of the particular form of entangled states.

2. Bell's inequalities for spin-1/2 and spin-1 entangled states of parallel spin-polarization and their violations

2.1 Spin-1/2 state

The violation of BIs was theoretically verified long ago for spin-1/2 bipartite entanglement with arbitrary superposition of opposite spin-polarization states^{28, 29} in stead of the spin singlet in the original formulation of Bell. It was revisited in a recent paper²⁶ based on quantum probability statistics, which has advantage that both the BI and its violation can be formulated in a unified manner with the help of state density-operator.²⁶ We consider in the present paper a two-spin entangled state with parallel polarization such that

$$|\psi\rangle = c_+|+, +\rangle + c_-|-, -\rangle \quad (1)$$

where c_{\pm} are two arbitrary complex coefficients with the normalization condition $|c_+|^2 + |c_-|^2 = 1$ and $|\pm\rangle$ denote the usual spin-1/2 eigenstates ($\hat{\sigma}_z|\pm\rangle = \pm|\pm\rangle$). Without losing generality, the two arbitrary coefficients c_{\pm} can be parameterized as

$$c_+ = e^{i\eta} \cos \xi, \quad c_- = e^{-i\eta} \sin \xi,$$

with ξ and η being two real parameters. The density-operator $\hat{\rho}$ of entangled state Eq.(1) can be split into two parts

$$\hat{\rho} = \hat{\rho}_{lc} + \hat{\rho}_{nlc}$$

in order to see the non-local quantum correlation explicitly. Here, the first part is a density operator of the complete mixed-state

$$\hat{\rho}_{lc} = \cos^2 \xi |+, +\rangle\langle +, +| + \sin^2 \xi |-, -\rangle\langle -, -|,$$

which describes two particles obeying the local or classical statistics in the absence of quantum interference at all. While, the quantum interference term

$$\hat{\rho}_{nlc} = e^{2i\eta} \sin \xi \cos \xi |+, +\rangle\langle -, -| + e^{-2i\eta} \sin \xi \cos \xi |-, -\rangle\langle +, +|$$

denotes the non-local correlation, which remains even if the two particles are separated in a space-like interval. Following Bell, two spins are measured independently along arbitrary directions respectively, say \mathbf{a} and \mathbf{b} . According to the quantum measurement principle, measuring outcomes fall into the eigenvalues of projection spin-operators $\hat{\sigma} \cdot \mathbf{a}$ and $\hat{\sigma} \cdot \mathbf{b}$, i.e. $\hat{\sigma} \cdot \mathbf{a}|\pm \mathbf{a}\rangle = \pm|\pm \mathbf{a}\rangle$, and $\hat{\sigma} \cdot \mathbf{b}|\pm \mathbf{b}\rangle = \pm|\pm \mathbf{b}\rangle$. Two orthogonal eigenstates of a projection spin-operator $\hat{\sigma} \cdot \mathbf{r}$ can be found explicitly as

$$|+\mathbf{r}\rangle = \cos\left(\frac{\theta_r}{2}\right)|+\rangle + \sin\left(\frac{\theta_r}{2}\right)e^{i\phi_r}|-\rangle$$

and

$$|-\mathbf{r}\rangle = \sin\left(\frac{\theta_r}{2}\right)|+\rangle - \cos\left(\frac{\theta_r}{2}\right)e^{i\phi_r}|-\rangle$$

which are called the spin coherent states of north- and south- pole gauges.³⁰ Here, $\mathbf{r} = (\sin \theta_r \cos \phi_r, \sin \theta_r \sin \phi_r, \cos \theta_r)$ with $\mathbf{r} = \mathbf{a}, \mathbf{b}$ is a unit vector parameterized by the polar and azimuthal angles (θ_r and ϕ_r) in the coordinate frame with z -axis along the direction of the initial spin-polarization. Outcome-independent base vectors for two-particle measurements are seen to be the direct product of eigenstates for operators $\hat{\sigma} \cdot \mathbf{a}$ and $\hat{\sigma} \cdot \mathbf{b}$. We can arbitrarily label these four base vectors as²⁶

$$|1\rangle = |+\mathbf{a}, +\mathbf{b}\rangle, |2\rangle = |+\mathbf{a}, -\mathbf{b}\rangle, |3\rangle = |-\mathbf{a}, +\mathbf{b}\rangle, |4\rangle = |-\mathbf{a}, -\mathbf{b}\rangle. \quad (2)$$

Then the correlation probability for independent measurements of two spins is the quantum statistical average of the correlation operator

$$\hat{\Omega}(ab) = (\hat{\sigma} \cdot \mathbf{a})(\hat{\sigma} \cdot \mathbf{b})$$

in the state $\hat{\rho}$

$$P(ab) = \text{Tr}[\hat{\Omega}(ab)(\hat{\rho}_{lc} + \hat{\rho}_{nlc})] = P_{lc}(ab) + P_{nlc}(ab), \quad (3)$$

Notice that the non-vanishing matrix elements of correlation operator in the outcome-independent base vectors Eq.(2) are obviously

$$\Omega_{11}(ab) = \Omega_{44}(ab) = 1$$

and

$$\Omega_{22}(ab) = \Omega_{33}(ab) = -1,$$

the correlation probability is found as

$$P(ab) = \rho_{11} + \rho_{44} - \rho_{22} - \rho_{33}.$$

The density-matrix elements denoted by $\rho_{ij} = \langle i | \hat{\rho} | j \rangle$, ($i, j = 1, 2, 3, 4$) can be split into two parts

$$\rho_{ii} = \rho_{ii}^{lc} + \rho_{ii}^{nlc},$$

with local elements given by

$$\rho_{11}^{lc} = \cos^2(\xi) K_a^2 K_b^2 + \sin^2(\xi) \Gamma_a^2 \Gamma_b^2$$

$$\rho_{22}^{lc} = \cos^2(\xi) K_a^2 \Gamma_b^2 + \sin^2(\xi) \Gamma_a^2 K_b^2$$

$$\rho_{33}^{lc} = \cos^2(\xi) \Gamma_a^2 K_b^2 + \sin^2(\xi) K_a^2 \Gamma_b^2$$

$$\rho_{44}^{lc} = \cos^2(\xi) \Gamma_a^2 \Gamma_b^2 + \sin^2(\xi) K_a^2 K_b^2.$$

Where $K_r^m = \cos^m(\theta_r/2)$ and $\Gamma_r^m = \sin^m(\theta_r/2)$ with $r = a, b, c, d$ denoting measurement directions and m the integer power-index throughout the paper. The non-local elements $\rho_{11}^{nlc}, \rho_{44}^{nlc}$ have equal value but opposite signs with respect to $\rho_{22}^{nlc}, \rho_{33}^{nlc}$

$$\begin{aligned} \rho_{11}^{nlc} &= \rho_{44}^{nlc} = -\rho_{22}^{nlc} = -\rho_{33}^{nlc} \\ &= \frac{1}{2} \sin \xi \cos \xi \sin \theta_a \sin \theta_b \cos(\phi_a + \phi_b - 2\eta). \end{aligned} \quad (4)$$

This property plays a crucial role, which we will see, in the violation of BIs. The measurement outcome correlation has the same form

$$P_{lc}(ab) = \text{Tr}[\hat{\Omega}(ab)\hat{\rho}_{lc}] = \cos \theta_a \cos \theta_b, \quad (5)$$

but a positive sign different from that for the spin singlet with opposite spin-polarizations²⁶ where the correlation formula has a negative sign seen from Eq.(4) of Ref.(26). This simple sign-difference leads to a necessary modification of the original formula of BI. In other words, the specific form of BI is state dependent. From the local correlation Eq.(5) it is easy to find the modified BI being

$$|P_{lc}(ab) - P_{lc}(ac)| \leq 1 - P_{lc}(bc), \quad (6)$$

different from the original²⁶ BI by a sign change in front of the local correlation $P_{lc}(bc)$ on the greater side of inequality. As a matter of fact, substitution of the correlation Eq.(5) into the less side of BI Eq.(6) yields

$$|P_{lc}(ab) - P_{lc}(ac)| \leq |\cos \theta_b - \cos \theta_c|.$$

Therefor it is straightforward to prove the modified BI in the parallel spin-polarization

$$|P_{lc}(ab) - P_{lc}(ac)| + P_{lc}(bc) \leq |\cos \theta_b - \cos \theta_c| + \cos \theta_b \cos \theta_c \leq 1.$$

The modified BI can be also verified by means of classical statistics following Bell as shown in Appendix. The non-local correlation

$$P_{nlc}(ab) = 2 \sin \xi \cos \xi \sin \theta_a \sin \theta_b \cos(\phi_a - \phi_b - 2\eta),$$

which comes from the quantum interference of coherent superposition of two-particle states, is responsible for the violation of BI. To see the violation of BI explicitly we assume that the parameters of superposition coefficients are $\xi = \pi/4$ and $\eta = n\pi$ with n being zero or integer. The total measurement correlation including the non-local parts becomes the well known quantum correlation-probability being a scalar product of the two unit-vectors \mathbf{a} and \mathbf{b}

$$P(ab) = \mathbf{a} \cdot \mathbf{b}, \quad (7)$$

which has a sign difference compared with the opposite spin-polarization.²⁶ The violation of BI has been investigated extensively²⁶ in terms of the quantum correlation-probability. The CHSH inequality remains not changed, since a common sign-difference of the measurement outcome correlation Eq.(5) for any two directions does not affect the absolute value

$$P_{CHSH}^{lc} = |P_{lc}(ab) + P_{lc}(ac) + P_{lc}(db) - P_{lc}(dc)|.$$

We still have

$$P_{CHSH}^{lc} \leq 2.$$

With the quantum correlation-probability Eq.(7) it is also obviously to have the well known formula

$$P_{CHSH} = |P(ab) + P(ac) + P(db) - P(dc)| \leq 2\sqrt{2}.$$

2.2 Spin-1 state

For the spin-1 entangled state of parallel spin-polarizations

$$|\psi\rangle = c_+|+1, +1\rangle + c_-|-1, -1\rangle,$$

the spin-coherent states of spin-1 projection operator $\hat{s} \cdot \mathbf{a}$ are found as²⁶

$$|+\mathbf{a}\rangle_1 = K_a^2|+1\rangle + \frac{1}{\sqrt{2}}e^{i\phi_a}\sin\theta_a|0\rangle + \Gamma_a^2e^{i2\phi_a}|-1\rangle$$

and

$$|-\mathbf{a}\rangle_1 = \Gamma_a^2|+1\rangle - \frac{1}{\sqrt{2}}e^{i\phi_a}\sin\theta_a|0\rangle + K_a^2e^{i2\phi_a}|-1\rangle,$$

where $\hat{s}_z|\pm 1\rangle = \pm|\pm 1\rangle$. The measurement-correlation probability of two spin-1 particles initially prepared in the entangled state with parallel spin-polarization can be obtained by the local matrix elements

$$\begin{aligned}\rho_{(1)11}^{lc} &= \cos^2(\xi)K_a^4K_b^4 + \sin^2(\xi)\Gamma_a^4\Gamma_b^4, \\ \rho_{(1)44}^{lc} &= \cos^2(\xi)\Gamma_a^4\Gamma_b^4 + \sin^2(\xi)K_a^4K_b^4, \\ \rho_{(1)22}^{lc} &= \cos^2(\xi)K_a^4\Gamma_b^4 + \sin^2(\xi)\Gamma_a^4K_b^4, \\ \rho_{(1)33}^{lc} &= \cos^2(\xi)\Gamma_a^4K_b^4 + \sin^2(\xi)K_a^4\Gamma_b^4.\end{aligned}$$

However the four non-local matrix elements are all equal

$$\rho_{(1)22}^{nlc} = \rho_{(1)33}^{nlc} = \rho_{(1)11}^{nlc} = \rho_{(1)44}^{nlc} = \frac{1}{8} \cos \xi \sin \xi \sin^2 \theta_a \sin^2 \theta_b \cos 2(\phi_a - \phi_b - \eta), \quad (8)$$

the same as in the case of antiparallel spin-polarizations.²⁶ It was explained in our previous paper²⁶ that the minus sign difference between $\rho_{11}^{nlc}, \rho_{44}^{nlc}$ and $\rho_{22}^{nlc}, \rho_{33}^{nlc}$ in Eq.(4) for the spin-1/2 is actually a nontrivial BP resulted from the relative reversal of two-spin measurements ($\rho_{22}^{nlc}, \rho_{33}^{nlc}$). In the spin-1 case the BP factor is only a trivial one $e^{i2\pi} = 1$. Therefore, the contributions of non-local interference between the same and opposite spin-polarization measurements cancel each other. The total outcome correlation is originated from the local density operator $\hat{\rho}_{(1)}^{lc}$ only, i.e.,

$$P_{(1)}(ab) = P_{(1)}^{lc}(ab) = \cos \theta_a \cos \theta_b.$$

There is no room for the violation of BIs in agreement with the previous observation for the spin-1 entangled state with antiparallel spin polarizations.²⁶ It is interesting to see a fact that the original form of BI is valid only for entangled states of antiparallel spin-polarizations. However our observation, that BIs are violated by the spin-1/2 but not the spin-1 entangled states, is true regardless of antiparallel or parallel polarizations. Now we extend our theorem to arbitrarily high spins.

3. Spin-parity phenomenon

For two spin- s particles the entangled macroscopic quantum-state (MQS) with parallel spin-polarization is defined as

$$|\psi\rangle = c_+|+s, +s\rangle + c_-|-s, -s\rangle. \quad (9)$$

The density operator of it can also be separated into the local part

$$\hat{\rho}_{(s)}^{lc} = \cos^2 \xi | +s, +s\rangle \langle +s, +s| + \sin^2 \xi | -s, -s\rangle \langle -s, -s|,$$

obeying the local or classical theory and the non-local part

$$\begin{aligned}\hat{\rho}_{(s)}^{nlc} &= e^{2i\eta} \cos \xi \sin \xi | +s, +s\rangle \langle -s, -s| \\ &+ e^{-2i\eta} \cos \xi \sin \xi | -s, -s\rangle \langle +s, +s|,\end{aligned}$$

respectively. Here, the extreme state $|\pm s\rangle$ ($\hat{s}_z|\pm s\rangle = \pm s|\pm s\rangle$) is called the MQS wherein the minimum uncertainty relation $|\langle \hat{s}_z \rangle| = 2\langle (\Delta \hat{s}_x)^2 \rangle^{1/2} \langle (\Delta \hat{s}_y)^2 \rangle^{1/2}$ is satisfied. So that the state $|\psi\rangle$ defined in Eq.(9) is called the Bell cat, which is actually entangled Schrödinger cat-states for both "dead" and both "life" cats. We assume that the measurements are restricted on MQS, namely the spin coherent states $|\pm \mathbf{a}\rangle_s$ with $\hat{s} \cdot \mathbf{a}|\pm \mathbf{a}\rangle_s = \pm s|\pm \mathbf{a}\rangle_s$. These spin coherent states can be generated from the extreme states $|\pm s\rangle$ such that

$$|\pm \mathbf{a}\rangle_s = \hat{R}|\pm s\rangle$$

with the generation operator

$$\hat{R} = e^{i\theta_a \mathbf{m} \cdot \hat{\mathbf{s}}}.$$

The unit-vector \mathbf{m} in the $x-y$ plane is perpendicular to the plane expanded by z -axis and the unit vector \mathbf{a} .

The explicit forms of spin coherent-states in the representation of Dicke states are given by^{30, 31}

$$|+\mathbf{a}\rangle_s = \sum_{m=-s}^s \binom{2s}{s+m}^{\frac{1}{2}} K_a^{s+m} \Gamma_a^{s-m} e^{i(s-m)\phi_a} |m\rangle,$$

$$|-\mathbf{a}\rangle_s = \sum_{m=-s}^s \binom{2s}{s+m}^{\frac{1}{2}} K_a^{s-m} \Gamma_a^{s+m} e^{i(s-m)(\phi_a+\pi)} |m\rangle.$$

In the outcome-independent base vector of two-particle measurements (corresponding to Eq. (2), however, with the spin-1/2 replaced by s ,) density-matrix elements for the Bell cat-state are given by:

$$\begin{aligned} \rho_{11}^{lc} &= \cos^2(\xi) K_a^{4s} K_b^{4s} + \sin^2(\xi) \Gamma_a^{4s} \Gamma_b^{4s}, \\ \rho_{22}^{lc} &= \cos^2(\xi) K_a^{4s} \Gamma_b^{4s} + \sin^2(\xi) \Gamma_a^{4s} K_b^{4s}, \\ \rho_{33}^{lc} &= \cos^2(\xi) \Gamma_a^{4s} K_b^{4s} + \sin^2(\xi) K_a^{4s} \Gamma_b^{4s}, \\ \rho_{44}^{lc} &= \cos^2(\xi) \Gamma_a^{4s} \Gamma_b^{4s} + \sin^2(\xi) K_a^{4s} K_b^{4s}, \end{aligned} \quad (10)$$

for the local part. With the above density-matrix elements Eq.(10) the normalized (the correlation per spin value) local correlations become

$$P_{(s)}^{lc}(ab) = (K_a^{4s} - \Gamma_a^{4s})(K_b^{4s} - \Gamma_b^{4s}), \quad (11)$$

which is also different from the opposite spin-polarization²⁶ by a negative sign compared with the Eq.(7) in Ref.(26). Using the corresponding local correlations Eq.(11) and notice $K_\alpha^{4s} - \Gamma_\alpha^{4s} \leq 1$ for $\alpha = a, b, c, d$, it is obviously that CHSH inequality remains the same

$$\left| P_{(s)}^{lc}(ab) + P_{(s)}^{lc}(ac) + P_{(s)}^{lc}(db) - P_{(s)}^{lc}(dc) \right| \leq 2.$$

While the BI is also modified as

$$1 - P_{(s)}^{lc}(bc) \geq |P_{(s)}^{lc}(ab) - P_{(s)}^{lc}(ac)|.$$

The non-local density-matrix elements are evaluated by

$$\rho_{11}^{nlc} = \rho_{44}^{nlc} = 2 \sin \xi \cos(\xi) K_a^{2s} \Gamma_a^{2s} K_b^{2s} \Gamma_b^{2s} \cos[2s(\phi_a + \phi_b - 2\eta)],$$

for the measurements along the same spin-polarization direction. The density-matrix elements for the measurements on opposite directions

$$\rho_{(s)22}^{nlc} = \rho_{(s)33}^{nlc} = (-1)^{2s} \rho_{(s)11}^{nlc}, \quad (12)$$

possess an additional geometric phase factor $e^{i2s\pi} = (-1)^{2s}$ compared with $\rho_{(s)11}^{nlc} = \rho_{(s)44}^{nlc}$. It was well explained in the previous paper²⁶ that the geometric phase or BP resulted from the relative reversal of spin-polarization measurements in the following matrix-element evaluation

$$\begin{aligned} \rho_{(s)22}^{nlc} &= e^{2i\eta} \sin \xi \cos(\xi) \langle +s | +\mathbf{a} \rangle \langle +\mathbf{a} | +s \rangle \langle -s | -\mathbf{b} \rangle \langle -\mathbf{b} | -s \rangle \\ &\quad + e^{-2i\eta} \sin \xi \cos(\xi) \langle -s | +\mathbf{a} \rangle \langle +\mathbf{a} | -s \rangle \langle +s | -\mathbf{b} \rangle \langle -\mathbf{b} | +s \rangle. \end{aligned} \quad (13)$$

Remarkably we come to the same conclusion as in the case of antiparallel spin-polarizations²⁶ that the non-local outcome correlation vanishes $P_{(s)}^{nlc}(a, b) = 0$ in the integer-spin Bell cat-state, since the BP factor is trivial and thus the four elements become equal ($\rho_{(s)11}^{nlc} = \rho_{(s)22}^{nlc}$) seen from Eq.(12). However, in the half-integer spin case an additional minus sign of the BP factor leads to $\rho_{(s)11}^{nlc} = -\rho_{(s)22}^{nlc}$, and the non-local part of correlation probability $P_{(s)}^{nlc}(ab)$ does not vanish any more. The spin parity phenomenon discovered in the previous paper²⁶ still exists for the Bell cat-states with parallel spin-polarization (both "dead" and both "life" cats) : the BI can be violated for the Bell cat-states of half-integer spins but not for the states of integer spins.

4. Conclusions and Discussions

The proposed formulation of quantum probability-statistics with state-density operator has advantage to separate the local and non-local measurement correlations. The non-local part, which comes from the quantum interference between two superposed state-components, can be evaluated independently to see why and how the violation of BIs takes place. For the bipartite-entanglement states of parallel spin-polarization the local measurement-correlations have only a sign difference with respect to the opposite spin-polarizations demonstrated in Ref.(26). The original BI is slightly modified due to the sign difference, however, CHSH inequality is not changed in the entangled state of parallel spin-polarization. The previous observation of spin-parity phenomenon²⁶ still holds, that the BIs are indeed violated for entangled MQS of half-integer spins but not the integer-spins. The violation for the half-integer spins can be understood as the effect of geometric phase induced by the relative reversal of spin measurements. We establish a relation between two-type of non-localities, namely, the violation of BI and the dynamic non-locality resulted from the geometrical phase.

Although the specific form of original BI depends on the initial state the spin-parity effect seems state independent. Our generic arguments of the spin-parity effect particularly for the nonviolation of BIs in the spin-1 entangled states can be tested by utilizing the orbital angular-momenta entanglement.³²⁻³⁴

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Appendix

Proof of the modified BI $|P_{lc}(ab) - P_{lc}(ac)| \leq 1 - P_{lc}(bc)$ for the entangled state with parallel spin-polarization in terms of classical statistics following Bell.

The correlation of product expectation-values for measuring two spins respectively along unit-vector directions \mathbf{a} , and \mathbf{b} is evaluated as

$$P(a, b) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda),$$

where $\rho(\lambda)$ is the probability distribution and λ is a hidden variable. $A(\mathbf{a}, \lambda) = \pm 1$ and $B(\mathbf{b}, \lambda) = \pm 1$ denote the outcome expectation-values of measurements for two spins. The measuring outcome expectation-values of two spins are equal $A(\mathbf{a}, \lambda) = B(\mathbf{b}, \lambda)$ for the entangled state Eq.(1) with parallel spin-polarization. Hence

$$P(a, b) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda),$$

and

$$p(a, b) - P(a, c) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda) [1 - A(\mathbf{b}, \lambda) A(\mathbf{c}, \lambda)].$$

It is obviously that

$$\begin{aligned} |P(a, b) - P(a, c)| &\leq \int d\lambda \rho(\lambda) [1 - A(\mathbf{b}, \lambda) A(\mathbf{c}, \lambda)] \\ &= 1 - P(b, c) \end{aligned}$$

since ρ is a normalized probability distribution $\int d\lambda \rho(\lambda) = 1$.

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